1. Homework Assignments

Dynamical Systems II

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`http://dynamics.mi.fu-berlin.de/lectures/`

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Problem 1: Consider the non-autonomous linear system $\dot{y}(t) = A(t)y(t)$, with periodic matrix A, $A(t+p) = A(t)$. Show that for any given initial time t_0, the Floquet theorem yields a decomposition of the Wronskian solution $y(t) = W(t,t_0)y(t_0)$ of the form

$$W(t,t_0) \;=\; Q_{t_0}(t)\mathrm{e}^{B_{t_0}(t-t_0)}$$

with constant complex matrix B_{t_0} and periodic complex matrix Q_{t_0}, $Q_{t_0}(t+p) = Q_{t_0}(t)$. How do Q_{t_0} and B_{t_0} depend on t_0 ?

Problem 2: Can an unstable equilibrium position become stable upon linearization? Can it become asymptotically stable? Can an asymptotically stable equilibrium become unstable?

Reminder: An equilibrium x is called stable under a flow Φ^t if for every $\varepsilon > 0$ there exists $\delta > 0$ such that the forward orbit of the δ-neighborhood of x under Φ^t remains a subset of the ε-neighborhood of x. The equilibrium x is called asymptotically stable if in addition there exists a neighborhood of x such that $\{x\}$ is the ω-limit set of each point in that neighborhood.

Problem 3: [Floquet-Theory for discrete dynamical systems] Consider the iteration

$$x_{k+1} = A_k x_k$$

with $A_{k+p} = A_k$ for all $k \in \mathbb{N}_0$ and fixed period $p \in \mathbb{N}$. Assume, that all complex matrices A_k are invertible.

Show, that there are complex matrices B, C_k, $k \in \mathbb{N}_0$ with $C_k = C_{k+p}$, $C_0 = \mathrm{Id}$ and

$$\prod_{k=0}^{m} A_k = C_m B^m \quad \forall m \in \mathbb{N}.$$

Problem 4: Annalyx gets the Floquet decomposition for the Wronskian $W(t,0)$ wrong. He writes $W(t,0) = \exp(Ct)R(t)$, instead of $W(t,0) = Q(t)\exp(Bt)$. Can he find a constant complex matrix C such that $R(t)$ becomes periodic, analogously to $B, Q(t)$ with $R(0) = \mathrm{Id}$? How are B, C and Q, R related? How are the Floquet multipliers and exponents affected?